Loop Transfer Matrix and Loop Quantum Mechanics

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Abstract

K Q_i, Q_f

Q

κ

d

d

d d

d

1 Introduction

R^d Z^d

$$T \cdot S\ area \quad \frac{}{\alpha} F\ extrinsic\ curvature\ ,$$

α $F\ ext\ curv\ \propto$

$$S \quad m \cdot A\ extrinsic\ curvature\ ,$$

m

$$A\ extrinsic\ curvature \quad \propto \quad length.$$

[1]

$$m\ A_{xy}\ extrinsic\ curvature \quad \rightarrow \quad m \int_x^y dl.$$

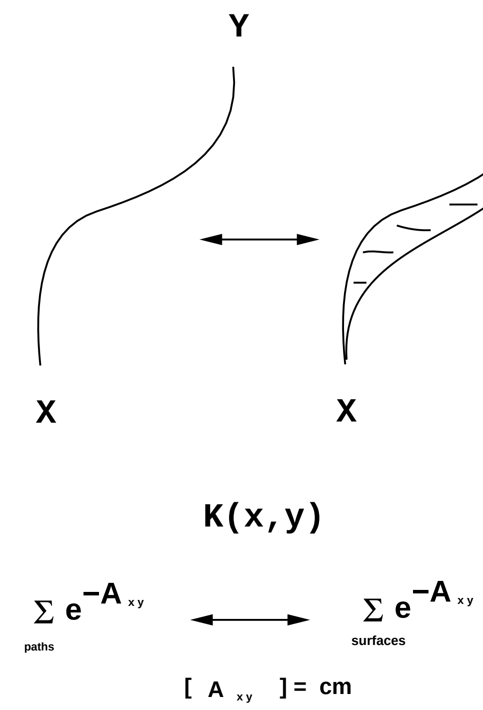
$$S_{eff} \quad m\ A\ extrinsic\ curvature \qquad T_{eff}\ S\ area \qquad ...,$$

T_{eff}

Z^d

d

[1]This is in contrast with the previous proposals when the extrinsic curvature term is a dimension-less functional $F(extrinsic\ curvature) \propto 1$ and can not provide this property.



A_{xy}

Z^3

$$K_{\kappa=0}\ Q_1, Q_2 \quad exp\{-\beta\ \ k\ Q_1 \quad l\ Q_1 \triangle Q_2 \quad k\ Q_2\ \},$$

Q_1 Q_2 $k\ Q$
$l\ Q$ $Q\ ^2$
Q_1 Q_2

κ

$$K\ Q_1, Q_2 \quad K_{\kappa=0}\ Q_1, Q_2\ \cdot exp\{-\ \kappa\beta\ \ k_{int}\ Q_1 \quad l\ Q_1 \overset{\leftrightarrow}{\cap} Q_2 \quad k_{int}\ Q_2\ \},$$

$k_{int}\ Q$ Q κ
$\kappa \to \infty,$

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[2] We shall use the word "loop" for the "polygon-loop".

$K(Q_1 \triangle Q_2)$

$Q_1 \triangle Q_2$

$_P$ Q $e^{i\pi s(P\cap Q)}$

loop momentum P s Q

Q

ψ_p x e^{ipx} $K(Q_1 \triangle Q_2)$

$$K(P_1, P_2) \quad \sum_{\{Q_1,Q_2\}} K(Q_1 \triangle Q_2) \; e^{i\pi s(P_1\cap Q_1) - i\pi s(P_2\cap Q_2)} \qquad _{P_1} \delta(P_1, P_2),$$

$_P$ P

$H(P) \quad -ln \quad P$. *loop quantum mechanics*

Q P

$H(P, Q)$

R^4

Z^4

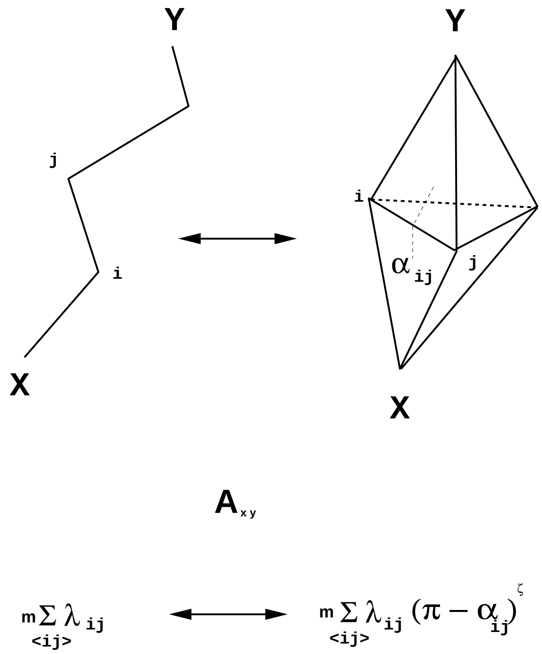
M

κ

$_P$

$$\frac{P_{\vec{r_1},\ldots\vec{r_n}} \; 3d}{\emptyset} \qquad < \sigma_{\vec{r_1}} \cdots \sigma_{\vec{r_n}} >_{2d} .$$

P $\vec{r_1}, \ldots \vec{r_n}$



X Y

2 Random Surfaces and Path Integral

Surfaces in Continuous Space.

$$m \sum_{<ij>} \lambda_{ij} \cdot |\pi - \alpha_{ij}|^{\zeta} \qquad \zeta \leq \ d - \ \ /d,$$

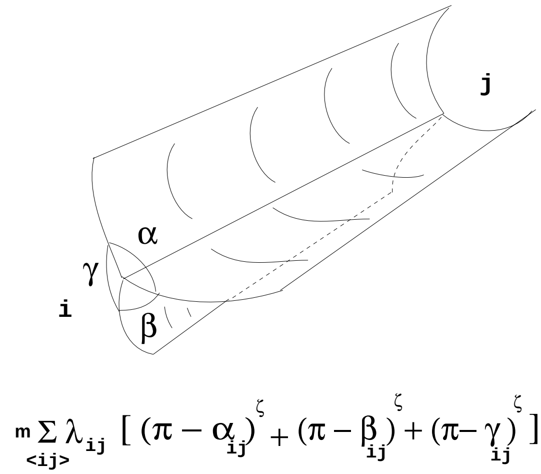
λ_{ij} $< ij >$ M α_{ij} M $< ij >$ [3]

$$\kappa \ m \sum_{<ij>} \lambda_{ij} \cdot \quad |\pi - \alpha^{1}_{ij}|^{\zeta} \quad ... \quad |\pi - \alpha^{r}_{ij}|^{\zeta} \quad .$$

r $< ij >$ r κ

$$A \ M \qquad m \sum_{<ij>} \lambda_{ij} \cdot |\pi - \alpha_{ij}|^{\zeta} \qquad m \ \kappa \sum_{<ij>} \lambda_{ij} \cdot \quad |\pi - \alpha^{1}_{ij}|^{\zeta} \quad ... \quad |\pi - \alpha^{r}_{ij}|^{\zeta} \quad .$$

[3]The angular factor defines the rigidity of the random surfaces and for $\zeta \leq (d-2)/d$ it increases sufficiently fast near angles $\alpha = \pi$ to suppress transverse fluctuations [6, 10].



α β γ π

κ

κ

[4]

$$\sum_{<ij>} \lambda_{ij} \cdot |\pi - \alpha|^{\varsigma} \quad \longleftrightarrow \quad \sum_{<ij>} \lambda_{ij}.$$

$\sigma_{classical}$ $A\ M \;\; \rightarrow m\ R \;\; T$.

$\sigma_{quantum}$ $\frac{d}{a^2}$ $- ln\frac{d}{\beta}$, d β a ς $\frac{d-2}{d}$ $\beta \rightarrow \beta_c$ d/e

$$A\ N \quad m \sum_{<i,j>} \lambda_{ij} \cdot |\ \pi - \omega_{ij}|^{\zeta}$$

λ_{ij} $<\ ij\ >$ N

$\pi - \omega_{ij}$ $<\ ij\ >$ ω_{ij} $<\ ij\ >$

[4]This property of the gonihedric action guarantees that spike instability does not appear here because the action is proportional to the total length of the spikes and suppresses the corresponding fluctuations [5, 6].

Equivalent spin system on a lattice [7].

$n_2 \; M$

M

$\pi/$

κn_4 $n_4 \; M$

κ

singular part

regular part

M

$$Z \;\; \beta \qquad \sum_{\{surfaces\;\; M\}} e^{-2\beta \;\; A(M)},$$

$A \;\; M$ M

$$A_{gonihedric} \;\; M \qquad n_2 \;\; M \qquad \kappa n_4 \;\; M \;\; .$$

$$H^{3d}_{gonihedric} \qquad - \;\; \kappa \sum_{\vec{r},\vec{\alpha}} \sigma_{\vec{r}}\sigma_{\vec{r}+\vec{\alpha}} \qquad \frac{\kappa}{} \sum_{\vec{r},\vec{\alpha},\vec{\beta}} \sigma_{\vec{r}}\sigma_{\vec{r}+\vec{\alpha}+\vec{\beta}} - \frac{-\;\kappa}{} \sum_{\vec{r},\vec{\alpha},\vec{\beta}} \sigma_{\vec{r}}\sigma_{\vec{r}+\vec{\alpha}}\sigma_{\vec{r}+\vec{\alpha}+\vec{\beta}}\sigma_{\vec{r}+\vec{\beta}},$$

$\vec{r}$ Z^3

$\vec{\alpha}$ $\vec{\beta}$

$H \;\; \kappa$

κ $\kappa \;\; /$

$. \;\; ^{N}$ N^3 κ

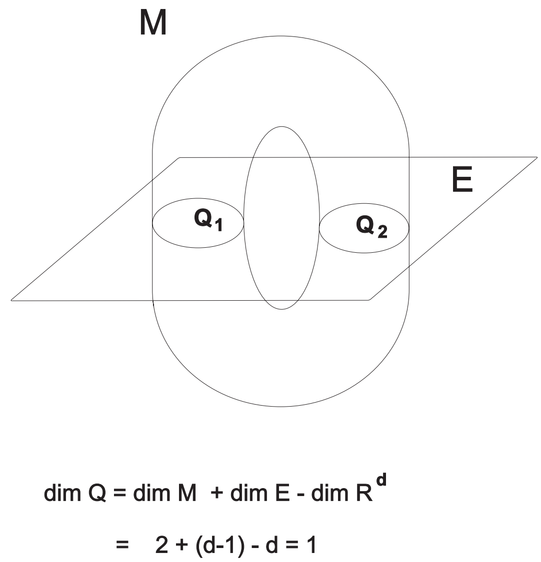
all states, including the ground state are exponentially degenerate ^{3N}

one can flip the spins on arbitrary layers, even on intersecting ones

Q

$l \;\; Q$

$k \;\; Q$



A M k E Q_1 E $, ..., Q_k$ E
E M
k E E
M

κ
κ $/$

3 Loop Transfer Matrix

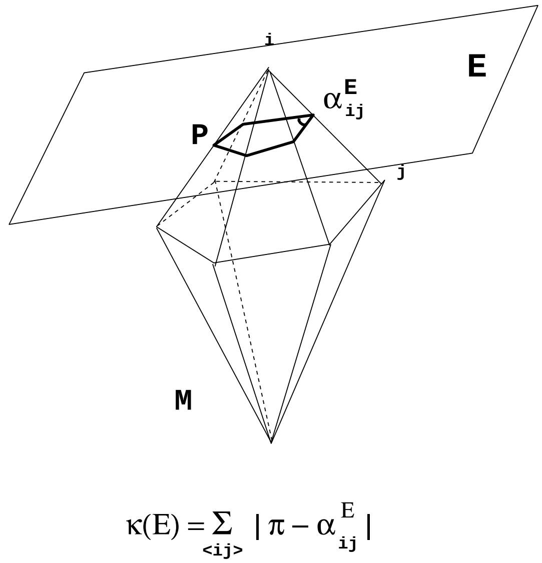
Geometrical Theorem [11].
κ

$$H^{3d}_{gonihedric} \; \kappa \qquad -- \sum_{\vec{r},\vec{\alpha},\vec{\beta}} \sigma_{\vec{r}}\sigma_{\vec{r}+\vec{\alpha}}\sigma_{\vec{r}+\vec{\alpha}+\vec{\beta}}\sigma_{\vec{r}+\vec{\beta}},$$

κ $/$

A M k E
E M
k E E
M
E M

$$Q_1 \; E \;, ..., Q_k \; E$$



M E
P k E

k E

$$k\ E \quad \sum_{i=1}^{k} k\ Q_i \quad \sum_{<i,j>} |\pi - \alpha^{E}_{ij}|,$$

α^{E}_{ij}
E $<\ ij\ >$

$Q_1\ E\ ,...,Q_k\ E$ $k\ E$ E
$A\ M$

$$A\ M \quad \frac{}{\pi}\int_{\{E\}} k\ E\ dE.$$

Geometrical Theorem on a lattice [12].
$A\ M$ Z^3 $\{E_x\},\{E_y\},\{E_z\}$
x,y,z Z^3_d
M M
$Q\ E$
M M
$k\ E$

$$A\ M \quad \sum_{\{E_x,E_y,E_z\}} k\ E\ .$$

$k\ E$
κ
$k\ E$

$$Z\ \beta \quad \sum_{\{M\}} exp\{-\ \beta \sum_{\{E\}} k\ E\ \}$$

$$exp\{- \beta \sum_{\{E\}} k E \} \quad \prod_{\{E\}} e^{-2\beta k(E)} \quad \prod_{\{E_z\}} e^{-2\beta k(E_z)} \prod_{\{E_y\}} e^{-2\beta k(E_y)} \prod_{\{E_x\}} e^{-2\beta k(E_x)}.$$

$\{E_z\}$

Transfer Matrix for κ *[12].* $\{E_z\}$ $k\ E_x$ $k\ E_y$ $\{E_y\}$ $\{E_x\}$ M E_z^i E_z^{i+1} Q_i M E_z^i Q_{i+1} M E_z^{i+1} $k\ E_x$ $k\ E_y$ E_z^i E_z^{i+1} Q_i Q_{i+1}

$$l\ Q_i \quad l\ Q_{i+1} \ - \ \cdot l\ Q_i \cap Q_{i+1} \quad l\ Q_i \triangle Q_{i+1} \ ,$$

$Q_1 \triangle Q_2 \ \equiv \ Q_1 \cup Q_2 \setminus Q_1 \cap Q_2$ $Q_1 \cup Q_2$ $Q_1 \cap Q_2$

$$A\ M \quad \sum_{\{E\}} k\ E \quad \sum_{\{E_z\}} k\ Q_i \quad l\ Q_i \triangle Q_{i+1}\ .$$

$$Z\ \beta \quad \sum_{\{Q_1,Q_2,\ldots,Q_N\}} K_\beta\ Q_1, Q_2\ \cdots K_\beta\ Q_N, Q_1 \quad tr K_\beta^N,$$

$K_\beta\ Q_1, Q_2$ $\gamma \times \gamma$

$$K_{\kappa=0}^{gonihedric}\ Q_1, Q_2 \quad exp\{-\beta\ \ k\ Q_1 \quad l\ Q_1 \triangle Q_2 \quad k\ Q_2\ \},$$

Q_1 Q_2 T^2 $N \times N$ γ N^2 T^2 Q_1 τ Q_2 τ [5] $k\ Q$ Q

$l\ Q$ Q $l\ Q_1 \triangle Q_2$ $\triangle$ Q_1 Q_2 Q $Q_1 \triangle Q_2$ [6] $l\ Q_1 \triangle Q_2$ Q_1 Q_2

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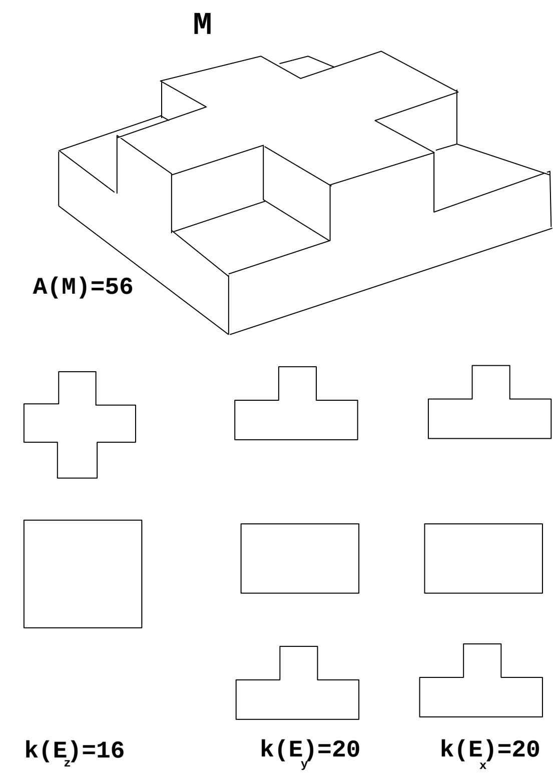
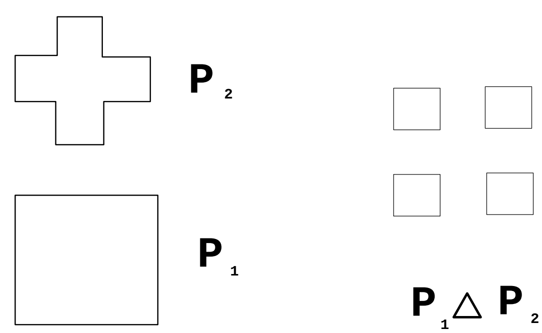
[5]Layer-to-layer transfer matrices for three-dimensional statistical systems have been considered in the literature [20, 21]. Using Yang-Baxter and Tetrahedron equations one can compute the spectrum of the transfer matrix in a number of cases [21]. In the given case the transfer matrix has geometrical interpretation which helps to compute the spectrum.

[6]Note that the operations $\cup$ and $\cap$ do not have this property. These operations acting on a polygon-loops can produce link configurations which do not belong to Π. The symmetric difference of sets $Q_1 \triangle Q_2$ is an important concept in functional analysis [22].



$\{E_x\}, \{E_y\}, \{E_z\}$ x, y, z
M
M P E
M M
k E
A $a\frac{\pi}{2}$



P_1 τ P_2 τ
k P_1 , k P_2 , l $\emptyset \triangle P_1$, l $P_1 \triangle P_2$, l $P_2 \triangle \emptyset$
A M
$a\frac{\pi}{2}$

$K_\beta \; Q_1, Q_2$

$$\sum_{\{Q_2\}} K_\beta \; Q_1, Q_2 \qquad Q_2 \qquad \beta \qquad Q_1 \;,$$

Q Q

H L^2

$$Z^{3d} \; \beta \qquad {}^{N}_{0} \quad \ldots \quad {}^{N}_{\gamma-1},$$

$$-\beta \; f_{3d} \; \beta \qquad {}_{N\to\infty} \; \overline{N^3} \; ln \; Z^{3d} \; \beta \qquad {}_{N\to\infty} \; \overline{N^2} \; ln \quad {}_{0}.$$

Q_i Q_f

t M/β

$$K \; Q_i, Q_f \qquad {}^{-M}_{0} \sum_{\{Q_1, Q_2, \ldots, Q_{M-1}\}} K_\beta \; Q_i, Q_1 \; \cdots K_\beta \; Q_{M-1}, Q_f \;,$$

${}_{0}$ $M \le N$

Loop Transfer Matrix for κ / .

κ

κ

M E

κ

$k \; E$

κ /

$$\sum_{\{E_z\}} k \; Q_i \qquad l \; Q_i \triangle Q_{i+1} \;.$$

E_z

E_z

$$\kappa \cdot \sum_{\{E_z\}} k_{int} \; Q_i \;,$$

$k_{int} \; Q$ Q

E_z

E_z

Q $\overrightarrow{Q} \to \overleftarrow{Q}$

Z_2 $\overrightarrow{Q}$ $\overleftarrow{Q}$

E_z^i E_z^{i+1}

$l\ Q_i \overset{\leftrightarrow}{\cap} Q_j$ $\cap$

$\leftrightarrow$

$$\kappa \cdot \sum_{\{E_z\}} l\ Q_i \overset{\leftrightarrow}{\cap} Q_j$$

$$\kappa \cdot \sum_{\{E_z\}} k_{int}\ Q_i \quad l\ Q_i \overset{\leftrightarrow}{\cap} Q_{i+1}\ .$$

$$A\ M \quad \sum_{\{E_z\}} k\ Q_i \quad l\ Q_i \triangle Q_{i+1} \quad \kappa \cdot\ k_{int}\ Q_i \quad l\ Q_i \overset{\leftrightarrow}{\cap} Q_{i+1} \quad .$$

$K_\beta\ Q_1, Q_2$

$\gamma \times \gamma$

$$K_\beta\ Q_1, Q_2 \qquad exp\{-\beta\ \ k\ Q_1 \quad l\ Q_1 \triangle Q_2 \quad k\ Q_2\ \ \} \cdot$$
$$exp\{-\ \kappa\beta\ \ k_{int}\ Q_1 \quad l\ Q_1 \overset{\leftrightarrow}{\cap} Q_2 \quad k_{int}\ Q_2\ \ \},$$

Q_1 Q_2 T^2

$N \times N$ γ T^2

$$\kappa \to \infty,$$

4 Loop Fourier transformation

$K\ Q_1, Q_2$

$k\ Q_1 \cup Q_2$

$$K\ Q_1 \triangle Q_2 \quad exp\{-\beta\ \ k\ Q_1 \triangle Q_2 \quad l\ Q_1 \triangle Q_2\ \ \},$$

$$K\ Q_1 \triangle Q_2 \qquad exp\{-\ \beta\ l\ Q_1 \triangle Q_2\ \ \}.$$

$Q_1 \triangle Q_2$

$$\sum_{\{Q_2\}} K_\beta\ Q_1 \triangle Q_2 \qquad Q_2 \qquad \beta \qquad Q_1\ .$$

Q_2 Q Q_1

Q $Q_1 \triangle Q_2$

$$\sum_{\{Q\}} K_\beta\ Q \qquad Q \triangle Q_1 \qquad \beta \qquad Q_1\ .$$

Q

$$Q \triangle Q_1 \qquad Q\ \cdot\ \ Q_1\ .$$

$$\sum_{\{Q\}} K_\beta\ Q \qquad Q \qquad \beta$$

$\emptyset$ $K\ Q_1 \triangle Q_2$

$\psi\ x\ \ y \qquad \psi\ x\ \cdot \psi\ y$

$$\int K_\beta\ x - y\ \psi\ y\ dx \qquad \lambda\ \beta\ \psi\ y\ , \qquad K_\beta\ x - y \qquad exp\ -\beta\ x - y\ ^2$$

$$\psi\ x \qquad e^{ipx}, \qquad \lambda_p\ x \qquad e^{-p^2/4\beta}.$$

Q Q_1

$$\emptyset \qquad ^2\ Q$$

Q $\emptyset$ $\emptyset$

$$^2\ Q \qquad \Rightarrow \qquad Q \qquad \pm\ .$$

$_P$ Q $e^{i\pi s(P\cap Q)}$

P $\pm$ s Q

Q

$Q_1 \triangle Q_2$ $\cap P$ $Q_1 \cap P$ $\triangle$ $Q_2 \cap P$

s Q_i s Q_{i+1} $-$ $\cdot s$ $Q_i \cap Q_{i+1}$ s $Q_i \triangle Q_{i+1}$

P *loop momentum*

Q

$$_{P_1} \cdot \quad _{P_2} \quad \sum_{\{Q\}} \quad _{P_1} Q \quad _{P_2} Q \quad \sum_{\{Q\}} e^{i\pi s(P_1\cap Q)+i\pi s(P_2\cap Q)} \quad \sum_{\{Q\}} e^{i\pi s((P_1\triangle P_2)\cap Q)}$$

s Q

$P_1 \triangle P_2$ $/$ $\emptyset$ γ

$P_1 \triangle P_2$ $\emptyset$

$$\sum_{\{Q\}} e^{i\pi s((P_1\triangle P_2)\cap Q)} \quad ^{N^2}\delta \; P_1, P_2 \; .$$

$\emptyset$ $\emptyset$

exp $i\pi s$ $P\cap Q$ P

$$\sum_{\{Q\}} \cdot e^{i\pi s((P\cap Q)} \quad , \quad if \quad P \; / \; \emptyset.$$

P γ $^{N^2}$

$$_P \quad \sum_{\{Q\}} e^{-i\pi s(P\cap Q)-\beta[k(Q)+2l(Q)]} \quad _P \quad \sum_{\{Q\}} e^{-i\pi s(P\cap Q)-2\beta l(Q)}.$$

$\sum_{\{Q\}} e^{-2\beta l(Q)}$

$\sum_{\{Q\}} e^{-\beta[2l(Q)+k(Q)]}$ Z^{2d}

$\emptyset$

$$\emptyset \quad Z^{2d} \quad \emptyset \quad Z^{2d-Ising}.$$

[7]

$$\frac{P}{\emptyset} \quad \sum_{\{Q\}} e^{-i\pi s(P\cap Q)-2\beta l(Q)}/Z^{2d-Ising} \;\equiv\; < e^{-i\pi s(P\cap Q)} >_Q$$

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[7] In what follows we shall consider the transfer matrix (30). Analogous formulas are valid for the transfer matrix (29) with curvature term $k(Q)$.

$$\frac{P_{\vec{r_1},...\vec{r_n}}}{\emptyset} \quad < \sigma_{\vec{r_1}} \cdots \sigma_{\vec{r_n}} >_{2dIsing} .$$

P $\vec{r_1}, ...\vec{r_n}$

P $\square$

$$\frac{\square}{\emptyset} \quad < \sigma_{\vec{r}} > \quad \mu \ \beta$$

N^2

P $\sqsubset\sqsupset$

$$\frac{\sqsubset\sqsupset}{\emptyset} \quad < \sigma_{\vec{r}} \sigma_{\vec{r+1}} > \quad - u \ \beta$$

N^2 N^2

$$P \quad \frac{}{N^2} \sum_{\{Q\}} e^{i\pi s(P \cap Q)} \quad Q$$

$$K \ P_1, P_2 \quad \sum_{\{Q_1,Q_2\}} K \ Q_1 \triangle Q_2 \ e^{i\pi s(P_1 \cap Q_1) - i\pi s(P_2 \cap Q_2)} \quad {}_{P_1} \sum_{\{Q_2\}} e^{i\pi s(P_1 \cap Q_2) - i\pi s(P_2 \cap Q_2)}$$
$$_{P_1} \delta \ P_1, P_2 \ .$$

P_i P_f

t M/β

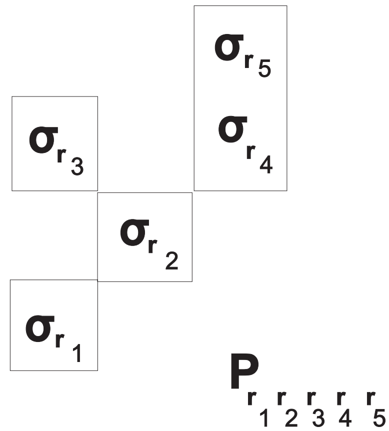
$$K \ P_i, P_f \quad \frac{P_i}{\emptyset}^{M} \ \delta \ P_i, P_f \quad e^{Mln(\frac{\Lambda_{P_i}}{\Lambda_\emptyset})} \ \delta \ P_i, P_f \quad e^{-MH(P_i)} \ \delta \ P_i, P_f \ ,$$

$$H \ P \quad -ln \ \frac{P}{\emptyset}$$

$H \ P$ P

Spin system which corresponds to the matrix $K \ Q_i \triangle Q_{i+1}$ *(30).*

$k \ Q_i$



$P_{\vec{r_1},\ldots\vec{r_n}}$ $\vec{r_1},\ldots,\vec{r_n}$

$P_{\vec{r_1},\ldots\vec{r_n}}$ n

$< \sigma_{\vec{r_1}} \cdots \sigma_{\vec{r_n}} >_{2d}$

z x y

$$\sum_{\{E_z\}} l \;\; Q_i \bigtriangleup Q_{i+1} \;\; .$$

$$H \qquad \sum_{over \;\; all \;\; edges} H_{edge},$$

$$H_{edge} \qquad U_1 U_{-1} \qquad U_1 U_{-1} \qquad \sigma_1\sigma_2\sigma_{-1}\sigma_{-2}.$$

z A_{edges} x y

E_x E_y x y

E_x E_y

$$H_{Q_1 \bigtriangleup Q_2} \qquad \sum_{E_x,E_y} \sigma\sigma\sigma\sigma.$$

E_z σ

λ μ

σ

$$\sum_i \sigma_i\sigma_{i+1}\lambda_i\lambda_{i+1} \qquad \sigma_i\sigma_{i+1}\mu_i\mu_{i+1} \qquad \sum_i \;\lambda_i\lambda_{i+1} \qquad \mu_i\mu_{i+1} \;\; \sigma_i\sigma_{i+1} \qquad \sum_i J_{eff}\sigma_i\sigma_{i+1},$$

$- \; , \; ,$ J_{eff} $/ \; , \; / \; , \; /$

$$J_{eff} \quad \lambda_i\lambda_{i+1} \quad \mu_i\mu_{i+1}.$$

E_z

J_{eff}

3d Ising Transfer Matrix.

$$K_\beta^{3d-Ising} \; P_1, P_2 \quad exp\{-\beta \;\; l \;\; P_1 \quad s \;\; P_1 \triangle P_2 \quad l \;\; P_2 \;\; \},$$

$s \;\; P$

$$curvature \; k \;\; P \;\; , \quad length \; l \;\; P \;\; , \quad area \; s \;\; P \;\; ,$$

$$K \;\; Q_1 \triangle Q_2 \quad exp\{-\beta \;\; l \;\; Q_1 \triangle Q_2 \quad s \;\; Q_1 \triangle Q_2 \;\; \}, \quad K \;\; Q_1 \triangle Q_2 \quad exp\{-\beta \;\; s \;\; Q_1 \triangle Q_2 \;\; \},$$

$$_P \quad \sum_{\{Q\}} e^{-i\pi s(P\cap Q)-\beta[l(Q)+2s(Q)]} \qquad _P \quad \sum_{\{Q\}} e^{-i\pi s(P\cap Q)-2\beta s(Q)}.$$

5 Matrices depending only on symmetric difference

$Q_1 \triangle Q_2$

$Q_1 \triangle Q_2$

$$Z^{3d} \;\; \beta \quad TrK^N \quad e^{-\beta f_{3d}(\beta)N^3}, \qquad K \;\; Q_1 \triangle Q_2 \quad e^{-\beta\Omega(Q_1 \triangle Q_2)}$$

$$_P \;\; Q \quad e^{i\pi s(P\cap Q)}.$$

$$_P \quad \sum_{\{Q\}} e^{-i\pi s(P\cap Q)} K \;\; Q \quad \sum_{\{Q\}} e^{-i\pi s(P\cap Q)-\beta\Omega(Q)}.$$

The eigenvalues $_P$ of three-dimensional system are exactly equal to the correlation functions of the two-dimensional system with the partition function

$$Z^{2d}\ \beta \quad \sum_{\{Q\}} e^{-\beta\Omega(Q)} \quad e^{-\beta f_{2d}(\beta)\cdot N^2} \quad \lambda_0^N \quad ... \quad \lambda_{2^N-1}^N,$$

$f_{2d}\ \beta$ λ_i

$\emptyset$

d

$$\emptyset \quad \sum_{\{Q\}} e^{-\beta\Omega(Q)} \equiv Z^{2d}\ \beta \quad \lambda_0^N \quad ... \quad \lambda_{2^N-1}^N,$$

$\emptyset$

$$-\beta\ f_{3d}\ \beta \quad {}_{N\to\infty} \overline{N^3}\ ln\ Z^{3d}\ \beta \quad {}_{N\to\infty} \overline{N^3}\ ln\{\ {}^N_\emptyset \quad ...\}$$
$${}_{N\to\infty} \overline{N^3}\ ln\{\ \lambda_0^N \quad ... \quad \lambda_{2^N-1}^N\ {}^N \quad ...\} \quad -\beta\ f_{2d}\ \beta\ .$$

$$f_{3d}\ \beta \quad f_{2d}\ \beta$$

$$\sum_{\{Q\}} e^{-i\pi s(P\cap Q)-\beta\Omega(Q)}/Z^{2d}\ \beta \quad < e^{-i\pi s(P\cap Q)} >_Q \quad < \sigma_{\vec{r_1}} \cdots \sigma_{\vec{r_n}} >_{2d},$$

$$\frac{P_{\vec{r_1},...\vec{r_n}}}{\emptyset} \quad < \sigma_{\vec{r_1}} \cdots \sigma_{\vec{r_n}} >_{2d}\ .$$

P $\vec{r_1},...\vec{r_n}$

$d-dimensional$
$Q_1 \triangle Q_2$

$d- \quad -dimensional$

Transfer Matrix for Membranes

M

$$K_\beta\ M_1, M_2 \quad exp\{-\beta\ \chi\ M_1 \quad A\ M_1 \triangle M_2 \quad \chi\ M_2\ \},$$

$\chi\ M$ M

$$\chi\ M \quad \sum_{<i>} |\ \pi - \omega_i|,$$

M $A\ M$

$$_P\ M \quad e^{i\pi v(P\cap M)},$$

$v\ M$ M

M_1 M_2

$$K\ M_1, M_2 \quad exp\{-\beta\ \chi\ M_1 \triangle M_2 \quad A\ M_1 \triangle M_2\ \}, \quad K\ M_1, M_2 \quad exp\{-\ \beta\ A\ M_1 \triangle M_2\ \},$$

$$_P \quad \sum_{\{M\}} e^{-i\pi v(P\cap M)-\beta[\chi(M)+2A(M)]}, \qquad _P \quad \sum_{\{M\}} e^{-i\pi v(P\cap M)-2\beta A(M)}.$$

$P \quad \emptyset$

$$_\emptyset \quad \sum_{\{M\}} e^{-2\beta A(M)}$$

$$f_{4d}\ \beta \quad f_{3d}\ \beta \quad f_{2d-Ising}\ \beta$$

$$\frac{_P}{_\emptyset}\ ^{4d} \quad Correlation\ functions\ ^{3d}.$$

6 Discussion

References